Evaluating Optical-Loss and Connectivity Effects on Q-Fly Performance in Distributed Fault-Tolerant Quantum Computing

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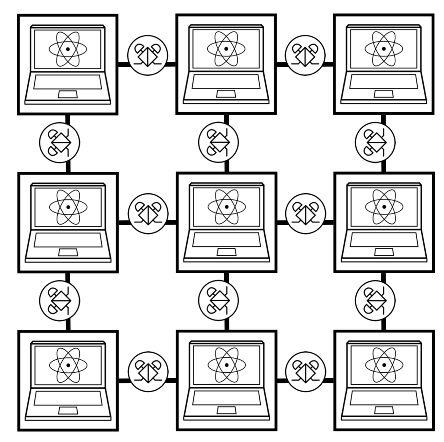
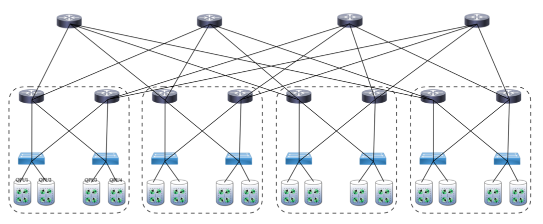
Introduction

Major Research Question

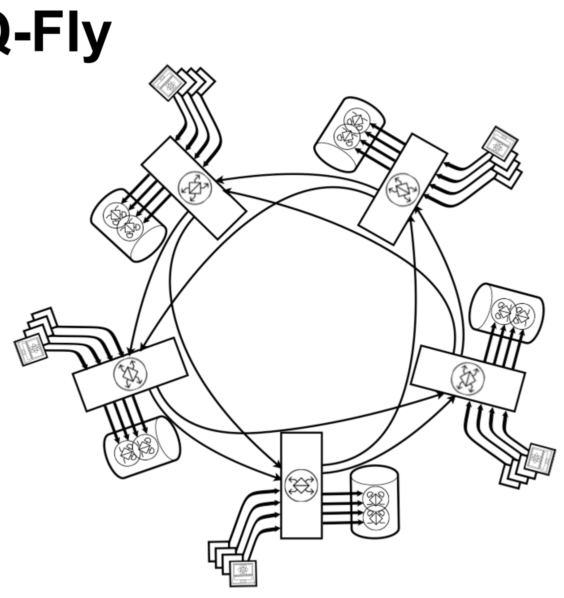
- What topology is effective or practical for quantum algorithms?
- How does it scale?

2D Grid **Fat-tree** **Q-Fly**



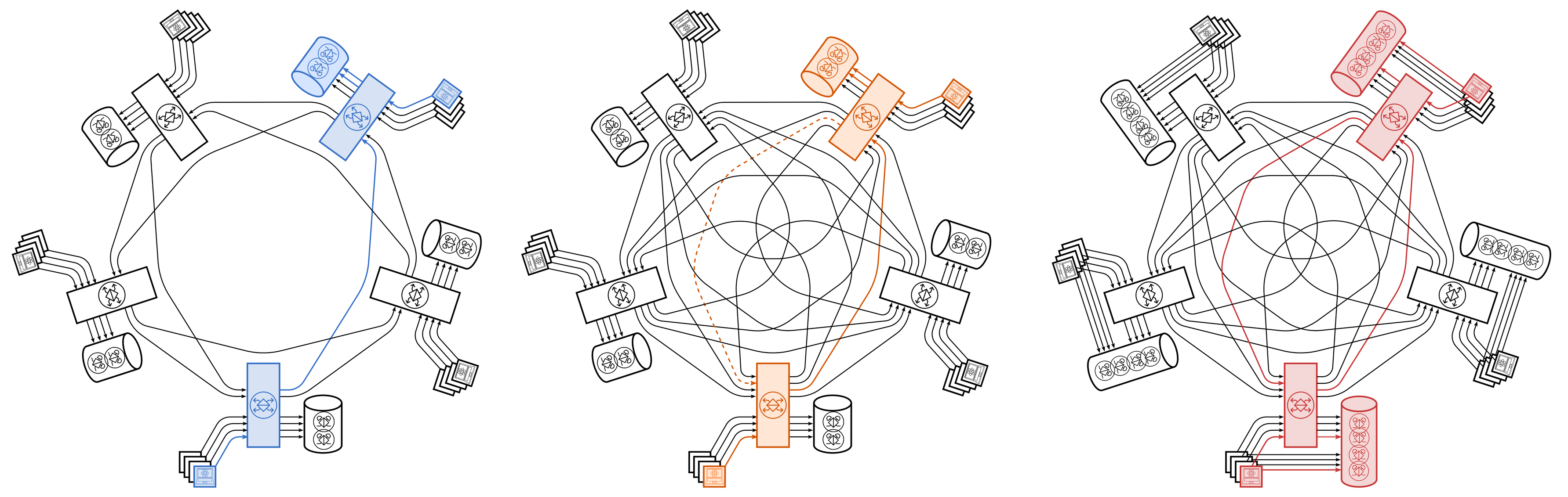


H.Shapourian et. al. arXiv:2501.05598



D.Sakuma et. al. arXiv:2412.09299

QPU BSA (Bell-State Analyzer) Optical switch



Features
- Provides fewer hops across arbitrary node pair:
- Parameters: g (# of groups), p (# of nodes), b (# of BSAs), loss

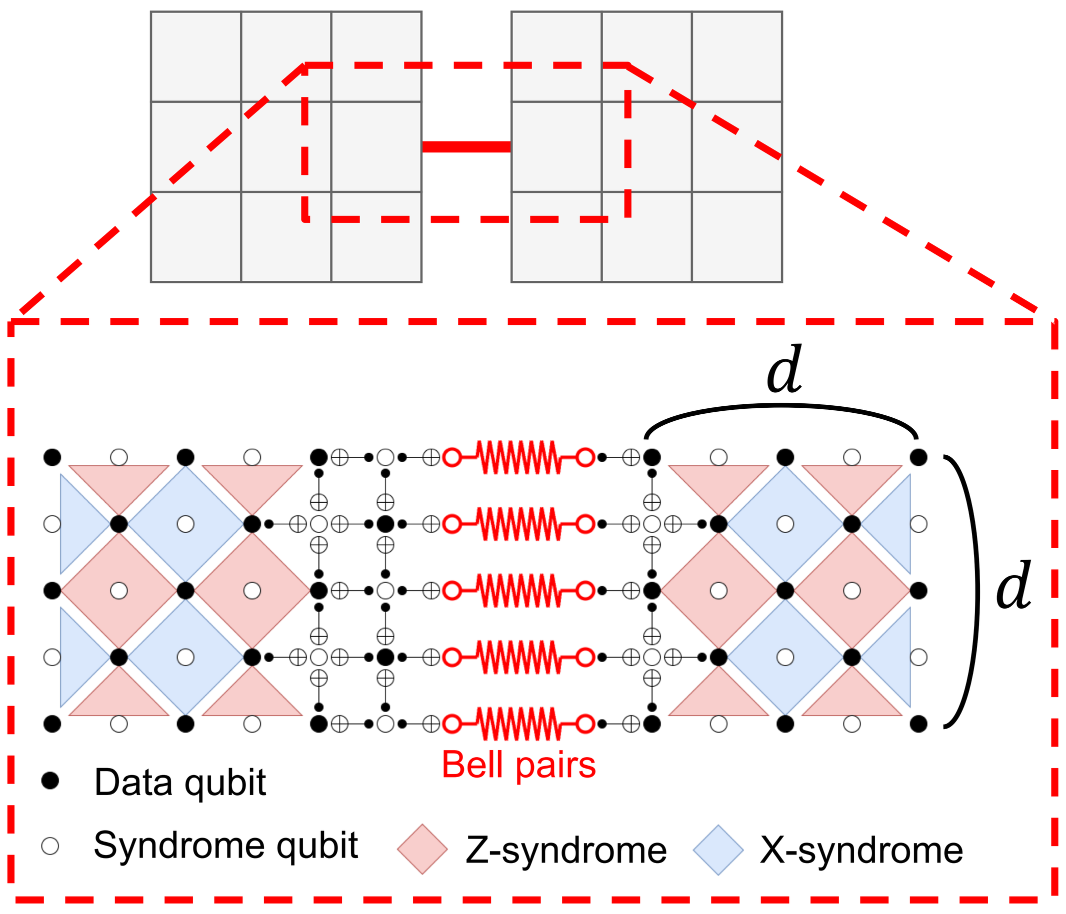
Question
- What is effective parameters and its span?
- How to benchmark its execution time?

Remote Gate Execution Model

Surface code patch
Fiber-based link

Remote lattice surgery based CX gate duration

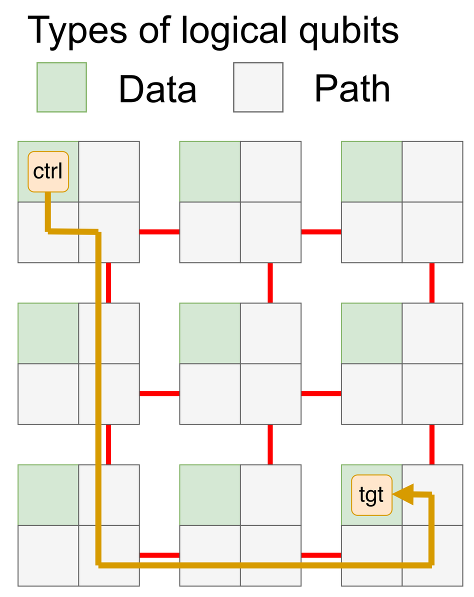
$$\tau_{\text{remote-CX}} = \frac{N_{\text{req}}\; d\times(2d-1)}{r_{\text{trial}}\, P_{\text{Bell}}\,\eta_{\text{prop}}\,\eta_{\text{switch}}^{N_{hop}}} + \tau_{\text{local}}$$



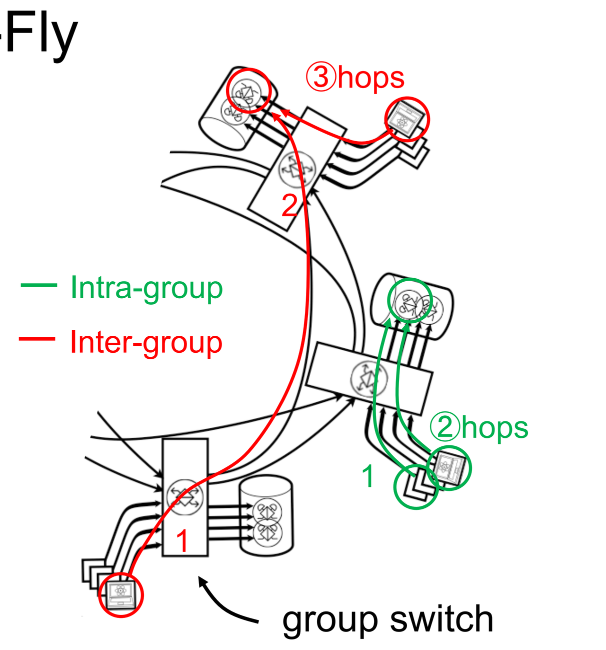
AG.Fowler et. al. PRL **104**, 180503 (2010)

Lattice surgery routing

2D Grid



Q-Fly



Trial rate for Bell pair generation : r_{trial}

Bell measurement success probability : P_{Bell}

Local operation time: τ_{local}

Required Bell pairs: N_{req}

Propagation transmittance: η_{prop}

Number of hops across end nodes: N_{hop}

Topology	N_{hop}	Actual form of $\eta_{\text{switch}}^{N_{hop}}$
Grid	Any	1
Q-Fly (SPHD)	2 (intra-group)	$\left(\eta_{2\times2}^{p+\lfloor g/2\rfloor}\right)^2$
	3 (inter-group)	$\left(\eta_{2\times2}^{p+\lfloor g/2\rfloor}\right)^3$
Q-Fly (DPHD)	2 (intra-group)	$\left(\eta_{2\times2}^{p+g}\right)^2$
	3 (inter-group)	$\left(\eta_{2\times2}^{p+g}\right)^3$
Q-Fly (DPFD)	1 (intra-group)	$\eta_{2\times2}^{p+g}$
	2 (inter-group)	$\left(\eta_{2\times2}^{p+g}\right)^2$

Group switch transmittance:

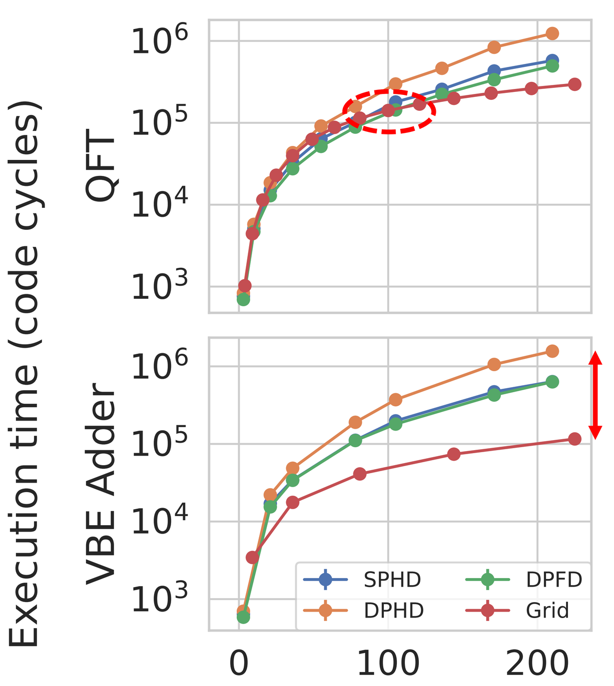
$$\eta_{\text{switch}} = \begin{cases} 1 & \text{(Grid: NO switch)} \\ \eta_{2\times2}^{k} & \text{(Q-Fly: Spank-Benes)} \end{cases}$$

RA Spanke & VE Benes Appl. Opt., vol. 26, no. 7, pp. 1226 (1987)

Radix: $k = \begin{cases} p+\lfloor g/2\rfloor & \text{(SPHD)} \\ p+g & \text{(DPHD, DPFD)} \end{cases}$

Transmittance of single elementary 2×2 switch : $\eta_{2\times2}$

Result1: Grid vs Q-Fly variants



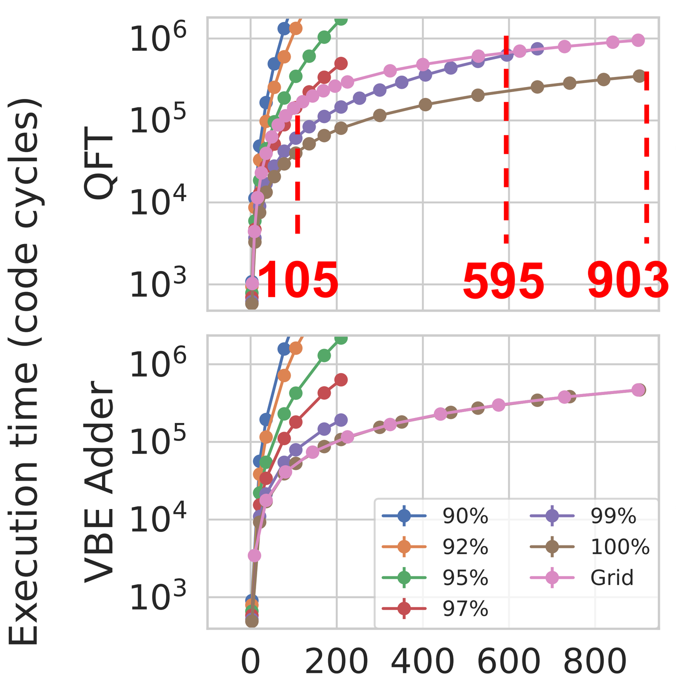
V.Vedral et. al. PRA **54**, 147 (1996)

condition: $\eta_{2\times2}$=0.97

[General Results]
- DPFD is the fastest in Q-Fly variants.
- DPHD increases path diversity rather than SPHD, but larger switch radix increases optical loss.
- DPFD reduces hop count while avoiding the DPHD switch-loss penalty.

[Benchmark Dependencies]
- QFT: Q-Fly superior up to 100 nodes.
- VBE: Grid has the best performance.

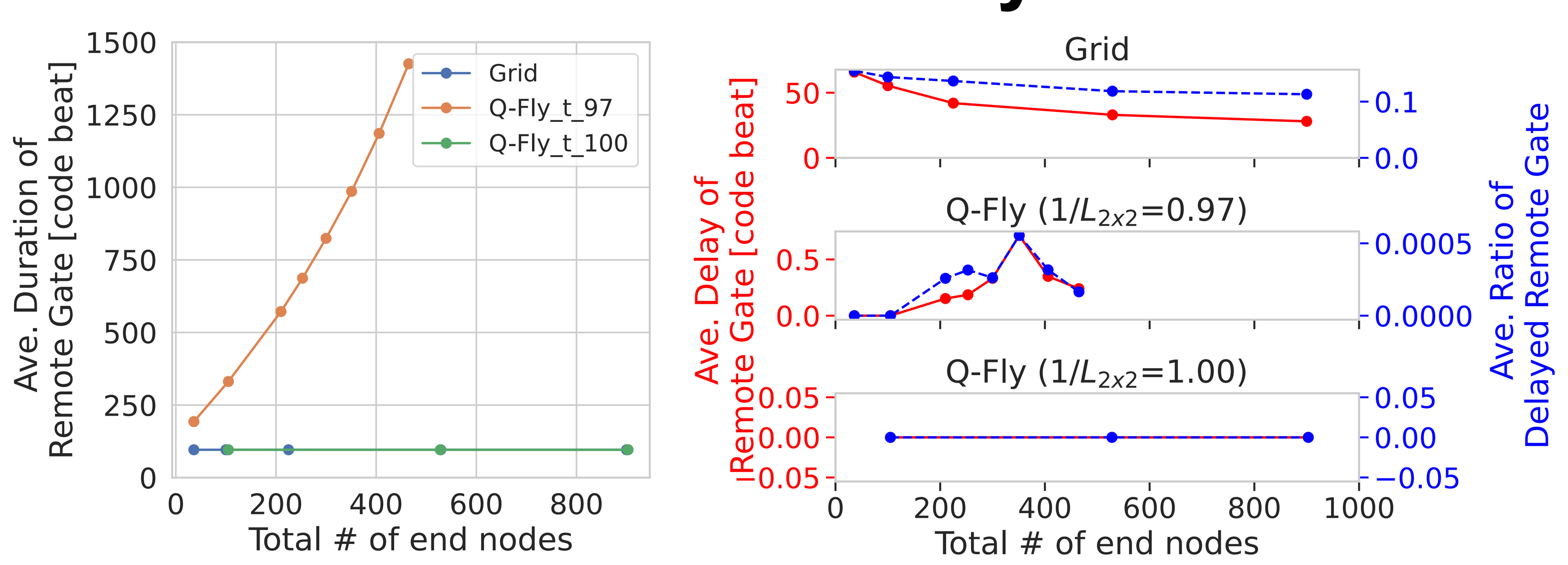
Result2: Transmittance Dependency in Q-Fly



[QFT]
Higher elementary-switch transmittance expands the range in which DPFD Q-Fly outperforms the 2D-grid: 105 nodes at 97%, 595 nodes at 99%, and at least 903 nodes with lossless switches.

[VBE Adder]
Q-Fly's performance approach Grid asymptotically with reducing $\eta_{2\times2}$. Q-Fly's connectivity itself has capability to achieve the same performance of optimized topology(Grid), however, impact of optical loss decrease Q-Fly's performance.

Discussion: Workload Analysis



- QFT communication in Q-Fly is not contention-limited; remote-gate queueing delay remains negligible.
- With finite switch loss, accumulated loss increases remote-gate service time at larger scales.
- Thus, Q-Fly's degradation is mainly a device-loss effect, requiring scalable low-loss optical switches.
- The 2D grid's average remote-gate delay decreases with size, likely due to more distributed path usage.

Conclusion

We evaluated execution-time dependence on quantum interconnect topology by introducing a remote-gate execution model that incorporates optical loss.

- DPFD Q-Fly is the fastest Q-Fly variant due to its lower hop count.
- Spanke–Benes group switches require >97% elementary-switch transmittance to outperform the 2D grid on QFT.
- Q-Fly connectivity suits QFT with negligible resource contention, and also asymptotically approach optimized Grid performance in VBE Adder.
- Accumulated optical loss increases remote-gate latency at larger scales, requiring scalable low-loss switch architectures.

Future Work

- Benchmark broader quantum workloads beyond QFT and VBE Adder.
- Evaluate QPU-size effects by varying logical data qubits per QPU.
- Extend the comparison to Fat-Tree and other interconnect topologies.
- Develop workload- and topology-aware compilation, mapping, routing, and scheduling methods.

Acknowledgement

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