Introduction to Quantum Computing
量子計算入門

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@Aizu U.

with help from
伊藤公平
阿部英介
and slides from T. Fujisawa (NTT)

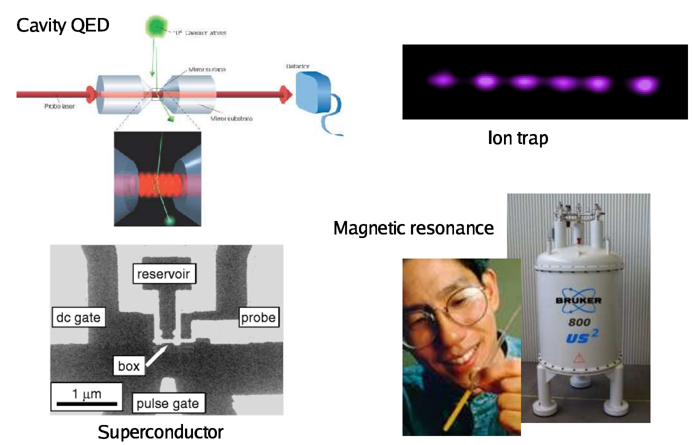
Course Outline

- Lecture 1: Introduction
- Lecture 2: Quantum Algorithms
- Lecture 3: Quantum Computational Complexity Theory
- Lecture 4: Devices and Technologies
- Lecture 5: Quantum Computer Architecture
- Lecture 6: Quantum Networking
- Lecture 7: Wrapup

量子計算の実行

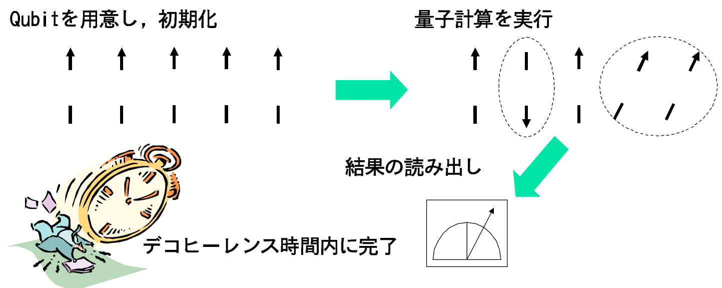
- IBM, Stanford, Berkeley, MIT (solution NMR)
- NEC (Josephson junction charge)
- Delft (JJ flux)
- NTT (JJ, quantum dot)
- 東大 (quantum dot, optical lattice, ...)
- 慶應 (silicon NMR, quantum dot)
- Caltech, Berkeley (quantum dot)
- Australia (ion trap, linear optics)
- Many others (cavity QED, Kane NMR, ...)

Physical Realization



DiVincenzo's Criteria

1. Well defined extensible qubit array
2. Preparable in the "000…" state
3. Long decoherence time
4. Universal set of gate operations
5. Single quantum measurements



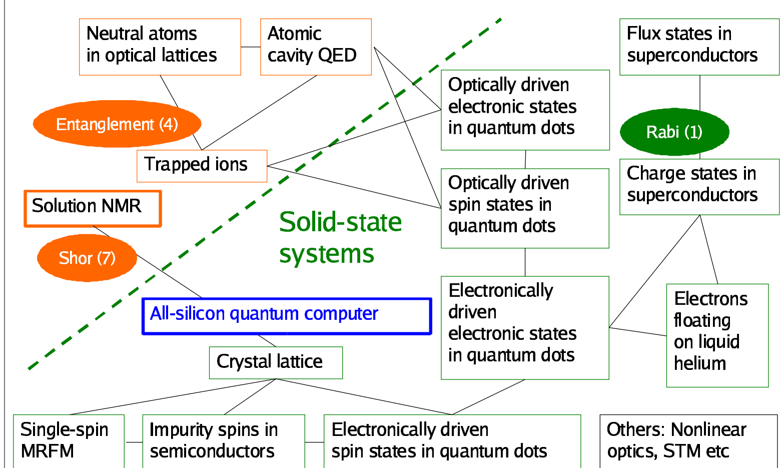
Qubit Representations

- Electron: number, spin, energy level
- Nucleus: spin
- Photon: number, polarization, time, angular momentum, momentum (energy)
- Flux (current)
- Anything that can be quantized and follows Schrodinger's equation

Problems

- Coherence time
  - nanoseconds for quantum dot, superconducting systems
- Gate time
  - NMR-based systems slow (100s of Hz to low kHz)
- Gate quality
  - generally, 60-70% accurate
- Interconnecting qubits
- Scaling number of qubits
  - largest to date 7 qubits, most 1 or 2

量子コンピュータの提案

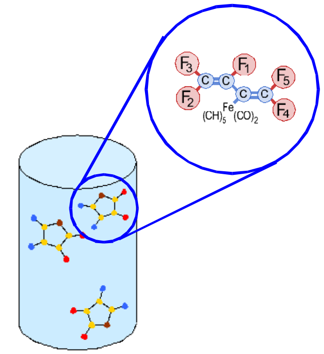


Technologies Reviewed

- Liquid NMR
- Solid-state NMR
- Quantum dots
- Superconducting Josephson junctions
- Ion trap
- Optical lattice
- All-optical

Liquid Solution NMR

Billions of molecules are used, each one a separate quantum computer. Most advanced experimental demonstrations to date, but poor scalability as molecule design gets difficult and SNR falls. Qubits are stored in nuclear spin of flourine atoms and controlled by different frequencies of magnetic pulses.
Used to factor 15 experimentally.

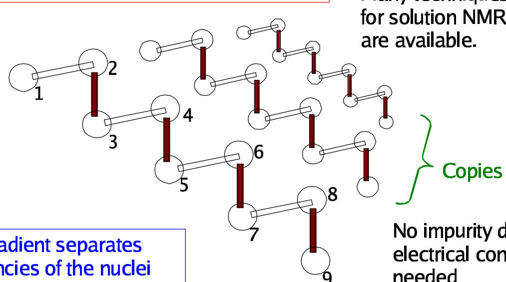


Vandersypen, 2000

All-Silicon Quantum Computer

10^5 ^{29}Si atomic chains in ^{28}Si matrix work like molecules in solution NMR QC.

Many techniques used for solution NMR QC are available.



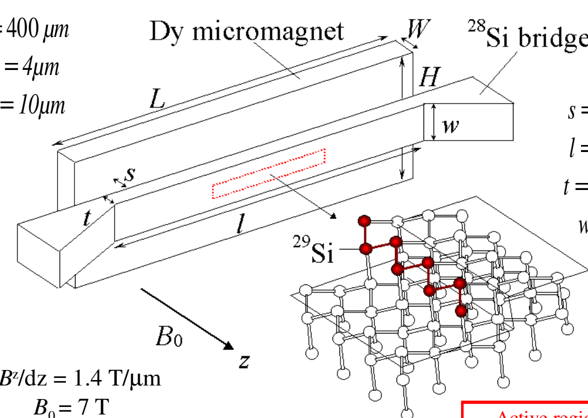
No impurity dopants or electrical contacts are needed.

A large field gradient separates Larmor frequencies of the nuclei within each chain.

T.D.Ladd *et al.*, Phys. Rev. Lett. **89**, 017901 (2002)

Overview

$L = 400\ \mu m$
$W = 4 \mu m$
$H = 10 \mu m$



$s = 2.1\ \mu m$
$l = 300 \mu m$
$t = 0.25\ \mu m$
$w = 4 \mu m$

$dB^z/dz = 1.4$ T/µm
$B_0 = 7$ T

Active region
100µm x 0.2µm

Keio Choice of System and Material

Two incompatible conditions to realize quantum computers:

2. Isolation of qubits from the environment
3. Control of qubits through interactions with the environment

System: NMR

1. Weak ensemble measurement
2. Established rf pulse techniques for manipulation

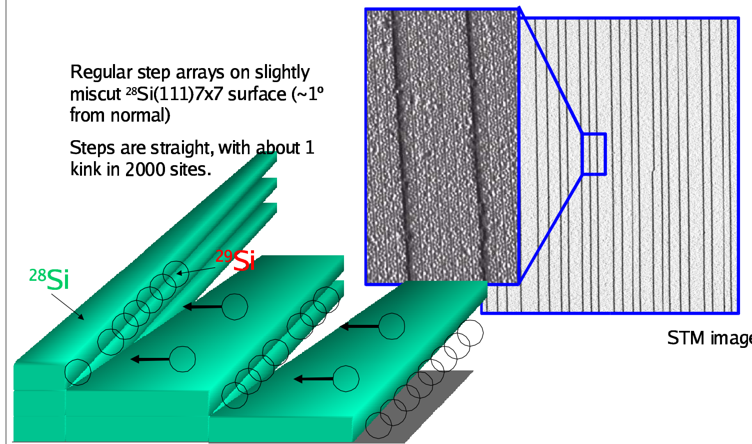
Material: silicon

1. Longest possible decoherence time
2. Established crystal growth, processing and isotope engineering technologies

Fabrication: ^{29}Si Atomic Chain

Regular step arrays on slightly miscut ^{28}Si(111)7x7 surface (~1° from normal)

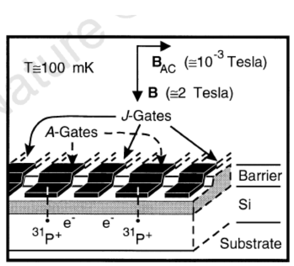
Steps are straight, with about 1 kink in 2000 sites.



J.-L.Lin *et al.*, J. Appl. Phys **84**, 255 (1998)

Kane Solid-State NMR

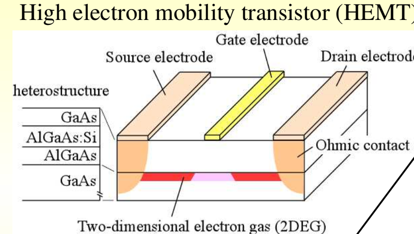
Qubits are stored in the spin of the nucleus of phosphorus atoms embedded in a zero-spin silicon substrate. Standard VLSI gates on top control electric field, allowing electrons to read nuclear state and transfer that state to another P atom.

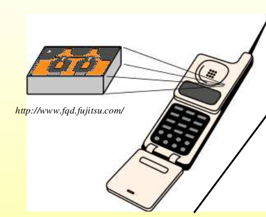


Kane, Nature, 393(133), 1998

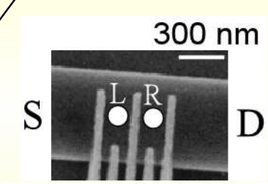
Semiconductor nanostructure

High electron mobility transistor (HEMT)

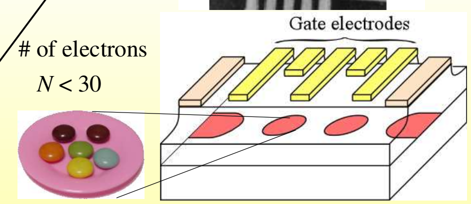




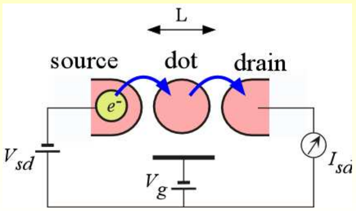
Quantum dot structure

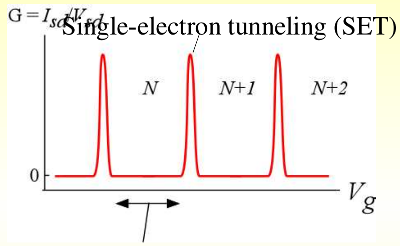


# of electrons
$N < 30$

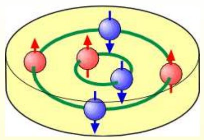


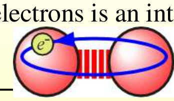
Quantum dot in the Coulomb blockade regime





Coulomb blockade region,
(the number of electrons is an integer,

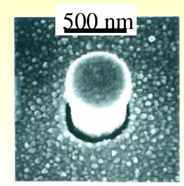




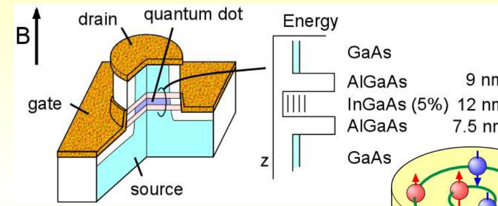
orbital degree of freedom
&
spin degree of freedom
(**spin qubit**)

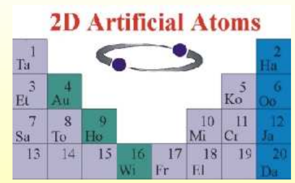
double quantum dot
bonding & anti-bonding orbitals
(**charge qubit**)

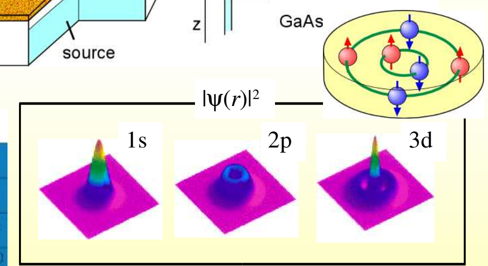
Quantum dot artificial atom



SEM picture of mesa after gate metallization

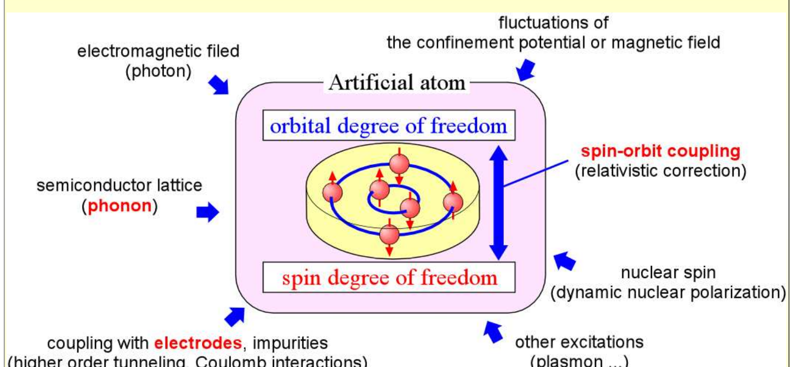






S. Tarucha et al., Phys. Rev. Lett. 77, 3613 (1996).

Environment surrounding a QD

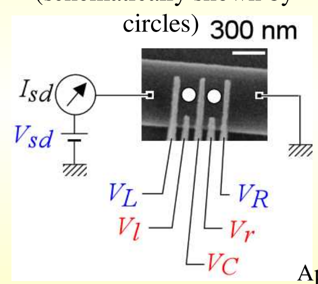


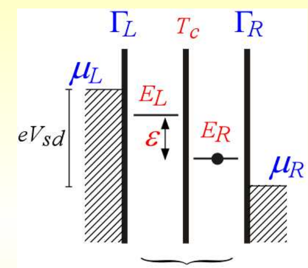
dissipation (T_1), decoherence (T_2) & manipulation

coherency of the system

Double quantum dot device

A double quantum dot fabricated in AlGaAs/GaAs 2DEG
(schematically shown by circles)





$H = \frac{1}{2}\varepsilon\sigma_z + T_c\sigma_x$

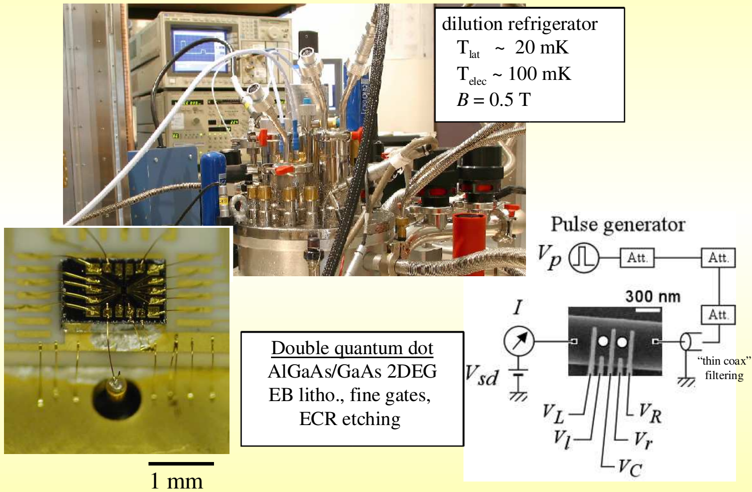
two-level system
(charge qubit)

cf. CPB charge qubit
(Y. Nakamura '99)

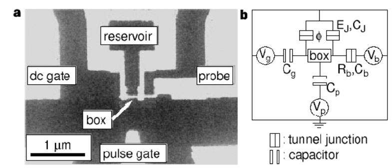
Approximate number of electrons: 10 ~ 30
Charging energy of each dot: ~ 1 meV
Typical energy spacing in each dot: ~100 ueV
Electrostatic coupling energy:~200 ueV

Lattice temp. ~ 20 mK (2 ueV)
Electron temp. ~ 100 mK (9 ueV)

Measurement system

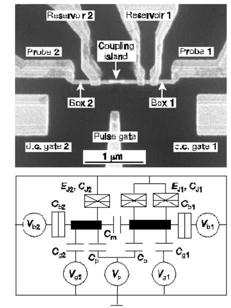


Josephson Junction Charge (NEC)



One-qubit device can control the number of Cooper pairs of electrons in the box, create superposition of states. Superconducting device, operates at low temperatures (30 mK).

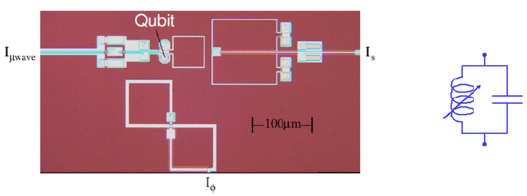
Nakamura et al., Nature, 398(786), 1999

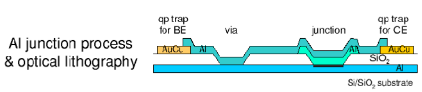


Two-qubit device

Pashkin et al., Nature, 421(823), 2003

JJ Phase (NIST, USA)





Qubit representation is phase of current oscillation.
Device is physically large enough to see!

J. Martinis, NIST

JJ Flux (Delft)

The qubit representation is a quantum of current (flux) moving either clockwise or counter-clockwise around the loop.

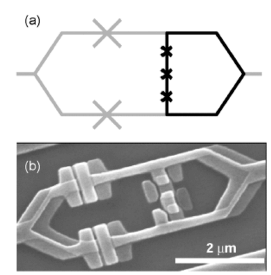
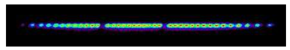
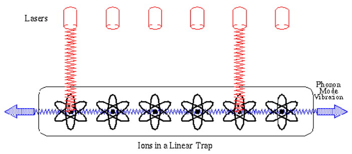


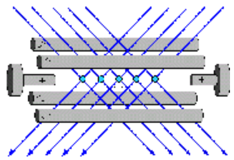
Fig. 1 Three-junction flux qubit; (a) Schematics. The gray part is for the readout; (b) Scanning-electron micrograph. The larger loop with two big junctions is a SQUID for readout.

Ion Trap

Ions (charged atoms) are suspended in space in an oscillating electric field. Each atom is controlled by a laser.



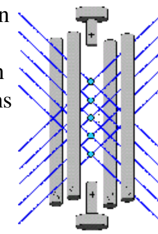


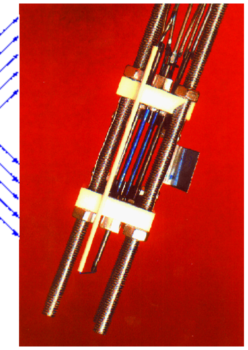


NIST, Oxford, Australia, MIT, others

Ion Trap

Number of atoms that can be controlled is limited, and each requires its own laser. Probably <100 ions max.

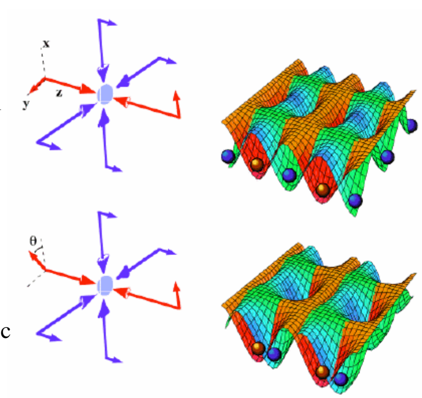




NIST, Oxford, Australia, MIT, others

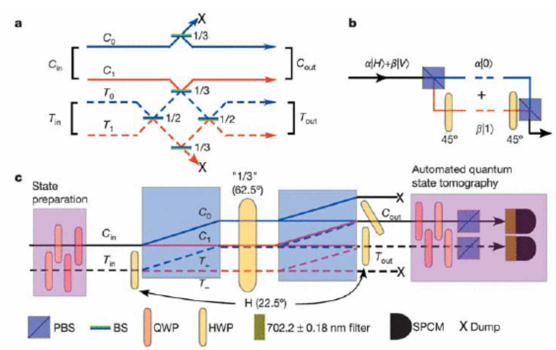
Optical Lattice (Atoms)

Neutral atoms are held in place by standing waves from several lasers.
Atoms can be brought together to execute gates by changing the waves slightly.
Also used to make high-precision atomic clocks.



Deutsch, UNM

All-Optical (Photons)



All-optical CNOT gate composed from beam splitters and wave plates.

O'Brien et al., Nature 426(264), 2003

Comparison

- NMR (Keio, Kane): excellent coherence times, slow gates
  - nuclear spin well isolated from environment
  - Kane complicated by matching VLSI pitch to necessary P atom spacing, and alignment
- Superconducting: fast gates, but fast decoherence
- Quantum dots: ditto
  - electrons in solid state easily influenced by environment

Comparison (2)

- Ion trap: medium-fast gates, good coherence time (one of the best candidates if scalability can be addressed)
- Optical lattice (atoms): medium-fast gates, good coherence time; gates and addressability of individual atoms need work
- All-Optical (photons): well-understood technology for individual photons, but hard to get photons to interact, hard to store

By the Numbers

Technology	Decoherence Time	Gate Time	Gates
All-Si NMR	25 seconds	high millisecs	10000?
Kane NMR	seconds	high millisecs	1000?
Ion Trap	seconds	low millisecs	1000?
Optical Lattice	seconds	low millisecs	?
Quantum Dot	low microsecs	nanosecs	100?
Josephson Junction	microseconds	nanosecs	10-10000?
All-Optical	N/A	N/A	N/A

Apples-to-apples comparison is difficult, and coherence times are rising experimentally.

Quantum Error Correction and the Threshold Theorem

Our entire discussion so far has been on "perfect" quantum gates, but of course they are not perfect.

Various "threshold theorems" have suggested that we need 10^4 to 10^6 gates in less than the decoherence time in order to apply *quantum error correction* (QEC). QEC is a big enough topic to warrant several lectures on its own.

Wrap-Up

- Qubits can be physically stored on electrons (spin, count), nuclear spin, photons (polarization, position, time), or phenomena such as current (flux); anything that is quantized and subject to Schrodinger's wave equation.

Wrap-Up (2)

- Many technologies depend on VLSI
- Most are one or two qubits
- Have not yet started our own Moore's Law doubling schedule
- Several years yet to true, controllable, multi-qubit demonstrations
- 10-20 years to a useful system?

Tomorrow

- Quantum computer architecture
  - how do you scale up? how do you build a computer out of this? what matters?
- Quantum networking
  - Quantum key distribution
  - Teleportation
- Wrap-Up and Review